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SEARCH THEORY

by

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SEARCH THEORY

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SEARCH THEORY

Search theory[1] came into being during World War II with the work of B. O. Koopman and his colleagues in the Antisubmarine Warfare Operations Research Group (ASWORG). ASWORG was directed by P. M. Morse and reported to Admiral Ernest King, Chief of Naval Operations and Commander in Chief, U. S. Fleet. Inspired by Morse, many of the fundamental concepts of search theory such as sweep width and sweep rate had been established by Spring of 1942. Since that time, search theory has grown to be a major discipline within the field of Operations Research. Its applications range from deep-ocean search for submerged objects to deep-space surveillance for artificial satellites.

The reader interested in the origins of search theory should consult Morse [29] and Koopman [24]. Early use of search theory to develop Naval tactics is discussed in Morse and Kimball [30]; excerpts from this work appear in [31].

For a modern account of search theory, the reader should consult Stone [41]. This book provides a rigorous development of the theory and includes an excellent bibliography and notes on previous research. Washburn's monograph [52] is also recommended.

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1. This article, in essentially the same form, was originally prepared for the Kotz-Johnson Encyclopedia of Statistical Sciences copywrited by John Wiley and Sons.

Since World War II, the principles of search theory have been applied successfully in numerous important operations. These include the 1966 search for a lost H-bomb in the Mediterranean near Palomares, Spain, the 1968 search for the lost nuclear submarine Scorpion near the Azores ~~[36]~~, and the 1974 underwater search for unexploded ordnance during clearance of the Suez Canal. The U. S. Coast Guard employs search theory in its open ocean search and rescue planning ~~[39]~~. Search theory is also used in astronomy ~~[47]~~, and in radar search for satellites ~~[34]~~. Numerous additional applications, including those to industry, medicine, and mineral exploration, are discussed in the proceedings ~~[15]~~ of the 1979 NATO Advanced Research Institute on Search Theory and Applications. Applications to biology ~~are given in [17] and [18],~~ and an ~~application to~~ machine maintenance and inspection is described in ~~[32]~~. the literature.

Further references to the literature are provided in the first section of this document. This is followed in the second section by an illustration of how search theory can be used to solve an optimal search problem.

1. REVIEW OF SEARCH THEORY LITERATURE

Work in search theory can be classified, at least in part, according to the assumptions made about measures of effectiveness, target motion, and the way in which search effort is characterized. This review is organized according to these criteria.

Measures of Effectiveness

Among the many measures of effectiveness that are used in search analysis, the most common are:

probability of detection,

expected time to detection,

probability of correctly estimating target "whereabouts," and

entropy of the posterior target location probability distribution.

Usually the objective of an optimal search is to maximize probability of detection with some constraint imposed on the amount of search effort available. For a stationary target, it is shown in Stone [41]

that when the detection function is concave or the search space and search effort are continuous, a plan which maximizes probability of detection in each of successive increments of search effort (incrementally optimal) will also be optimal for the total effort contained in the increments (totally optimal).

Moreover, for stationary targets, it is often theoretically possible to construct a "uniformly optimal" search plan. This is a plan for which probability of detection is maximized at each moment during its period of application. If a uniformly optimal search plan exists, then it will (1) maximize probability of detection over any period of application (i.e., be totally and incrementally optimal) and (2) minimize the expected time to detection. An example of such a plan (originally due to Koopman) is given in the last section.

In a "whereabouts" search, the objective is to correctly estimate the target's location in a collection of cells given a constraint on search cost. The searcher may succeed either by finding the target during search or by correctly guessing the target's location after search. These searches were first studied systematically by Kadane (see [21]). In many cases of interest, Kadane shows that the optimal whereabouts search consists of an optimal detection search among all cells exclusive of the cell with the highest prior target location probability. If the search fails to find the target, then one guesses that it is in the excluded highest probability cell.

More recently, Kadane and Stone [22] have considered whereabouts search in the context of moving targets. They show that the optimal whereabouts search plan may be found by solving a finite number of optimal detection search problems, one for each cell in the grid.

Consideration of entropy as a measure of effectiveness is useful in certain situations and can be used to draw a distinction between search and surveillance. For certain stationary target detection search problems with an exponential detection function, Barker [5] has shown that the search plan which maximizes the entropy of the posterior target location probability distribution conditioned upon search failure is the same as the search plan which maximizes probability of detection.

In a surveillance search, the objectives are usually more complex than in a detection search. For example, one may wish to correctly estimate target location at the end of a period of search in order to take some further action. In this case, detection before the end of the period can contribute to success but does not in itself constitute success. More general problems of this type are discussed by Tierney and Kadane [49], and they obtain necessary conditions for optimality when target motion is Markovian. In surveillance problems involving moving targets and false contacts where the time of terminal action may not be known in advance, Richardson [38] suggests allocating search effort to minimize expected entropy (maximize information).

Among other measures of effectiveness which are used in search, those based upon minimax criteria are of particular interest. Corwin [8] considers search as a statistical game and seeks estimates for target location. Alpern [3], Gal [14], and Isaacs [19] consider games in which minimax strategies are sought for a moving target seeking to avoid a moving searcher.

Target Motion

Assumptions about target motion have a considerable influence on the characteristics of search plans and the difficulty of computation. Until recently all but the simplest search problems involving target motion were intractable from the point of view of mathematical optimization. Results were usually obtained by considering transformations which would convert the problem into an equivalent stationary target problem (e.g., see Stone and Richardson [42], Stone [43], and Pursiheimo [35]. Representative early work on search with Markovian target motion is given in Pollock [33], Dobbie [12], and McCabe [28]. Hellman [16] investigates the effect of search upon targets whose motion is a diffusion process.

The first computationally practical solution to the optimal search problem for stochastic target motion involving a large number of cells and time periods is due to Brown [6]. For exponential detection functions, he found necessary and sufficient conditions for discrete time and space search plans, and provided an iterative method for optimizing search for targets whose motion is described by mixtures of discrete time and space Markov chains. Washburn [51] extended Brown's necessary conditions to the case of discrete search effort. Washburn [53] also provides a useful bound on how close a plan is to the optimal plan.

Very general treatments of moving target search are provided by Stone [44] and by Stromquist and Stone [46] allowing efficient numerical solution in a wide class of practical moving target problems; these include, for example, non-Markovian motion and non-exponential detection functions.

The existence of optimal search plans for moving targets is not to be taken for granted. L. K. Arnold has shown that there are cases where no allocation function satisfies the necessary conditions given in [46]. In his examples, there appear to be optimal plans, but they concentrate effort on sets of measure zero and are outside the class of search allocation functions usually considered. He also shows the existence of optimal plans whenever the search density is constrained to be bounded.

Search Effort

Search effort may be either discrete (looks, scans, etc.) or continuous (time, track length, etc.).

In problems involving discrete search effort, the target is usually considered to be located in one of several cells or boxes. The search consists of specifying a sequence of looks in the cells. Each cell has a prior probability of containing the target. A detection function b is specified, where $b(j,k)$ is the conditional probability of detecting the target on or before the kth look in cell j, given that the target is located in cell j. A cost function c is also specified, where $c(j,k)$ is the cost of performing k looks in cell j. An early solution to this problem for independent glimpses and uniform cost is given by Chew [7]. In this case,

$$b(j,k) - b(j,k-1) = \alpha_j(1-a_j)^{k-1}$$

for all j and for $k > 0$; $c(j,k) = k$ for all j and $k \geq 0$. Additional important results have been obtained by Matula [27], Blackwell (see Matula [27]), Kadane [20], and Wegener [54], [55], and [56].

Kadane's result [20] is particularly interesting since he uses a variant of the Neyman-Pearson lemma to obtain an optimal plan for the general case where $b(j,k) - b(j,k-1)$ is a decreasing function of k for all j.

In problems involving continuous effort, the target may be located in Euclidean n-space or in cells as in the case of discrete search. In the former case it is assumed that the search effort is "infinitely divisible" in the sense that it may be allocated as finely as necessary over the entire search space. The search problem was originally expressed in this form by Koopman (see [23]). The continuous effort case will be considered in greater detail in the remainder of this article.

Just as with discrete search effort, there is a detection function b, where $b(x,z)$ is the probability of detecting the target with z amount of effort applied to the point x, given the target is located at x. If x is a cell index, then z represents the amount of time or track length allocated to the cell. If x is a point in Euclidean n-space, then z is a density as will be made clear in the next section.

Koopman's original solution [23] to the search pr[illegible] made use of an exponential function for b of the form

$$b(x,z) = 1 - \exp(-\kappa z) \quad ,$$

where κ is a positive constant which may depend upon x. DeGuenin [10] considered a more general class of detection functions now referred to as "regular" (see [41]). Dobbie [11] considered sequential search with a concave detection function. Richardson and Belkin [37] have treated a special type of regular detection function obtained when the parameter κ in the exponential effectiveness function is a random variable. Such functions occur when sensor capabilities are uncertain. Tatsuno and Kisi [48] address similar problems. Stone has considered very general detection functions and has collected the results in [41].

For differentiable detection functions, Wagner [50] obtained sufficient conditions for an optimal search plan with continuous effort using a non-linear functional version of the Neyman-Pearson lemma.

Remarks

Search theory remains a field of active research in spite of the considerable advances made since its inception more than forty years ago. A review of the current status of search theory in terms of practical applications is given in [45]. Many problems remain to be solved, particularly in cases involving multiple targets and false targets. Also systematic methods are needed for constructing the prior target

location probability distribution from sometimes conflicting subjective opinion. More work is also needed on problems where it is essential to take exact account of search track continuity or the switching cost of moving from one region to another. These problems remain intractable although some recent progress has been made (see, e.g., [54] and [25]).

Brown's innovative solution to an important class of moving target search problems has removed an impediment to progress in this area. An interesting extension of Brown's results occurs in the context of search and rescue operations carried out at sea. Here the target's state may be unknown. That is, the target may be the survivor of a mishap and may be aboard the distress vessel, afloat in a life raft, or alone in the water supported only by a life jacket. The detectability of the target varies with these alternatives as does the drift motion and the expected time of survival of the target. A solution to this important problem is given in [9] under very general assumptions.

Unfortunately, Brown's approach applies only to the case where search effort is continuous. Efficient algorithms are still required in the case where search effort is discrete.

2. SOLUTION TO AN OPTIMAL SEARCH PROBLEM

This section shows how optimal search theory can be applied to an important class of search problems where the prior (i.e., before search) probability distribution for target location is normal and the detection function b (see section 1) is exponential. Many search problems which occur in practice are of this form.

Definitions

Let X denote the plane, and let A be some arbitrary region of interest. Then the prior probability that the target's location $\tilde{\mathbf{x}}$ is in A is given by

$$\Pr[\tilde{\mathbf{x}} \in A] = \int_A p(\mathbf{x})dx \quad ,$$

where

$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp(-|\mathbf{x}|^2/2\sigma^2) \tag{1}$$

is the circular normal density function, with mean at the origin and variance σ^2 in both oordinate directions. The distance of $\mathbf{x}$ from the origin is denoted by $|\mathbf{x}|$.

Let F be the class of non-negative functions defined on X with finite integral. By definition, this is the class of search allocation functions, and for $f \in F$

$$\int_A f(\mathbf{x})\, d\mathbf{x}$$

is the amount of search effort placed in region A.

We will assume that the unit of search effort is "time" and thus the "cost" C associated with the search is the total amount of time consumed. In this case $c(\mathbf{x},z) = z$, and the cost functional C is defined by

$$C[f] = \int_X c\big(\mathbf{x},\ f(\mathbf{x})\big)\, d\mathbf{x} = \int_X f(\mathbf{x})\, d\mathbf{x} \quad .$$

The measure of effectiveness will be probability of detection. Hence, the "effectiveness functional" D is assumed to have the form

$$D[f] = \int_X b\big(\mathbf{x},\ f(\mathbf{x})\big)\ p(\mathbf{x})\ dx \quad , \tag{2}$$

where b is the detection function. As mentioned earlier, the detection function is assumed to be exponential, and hence, for $R > 0$

$$b(\mathbf{x}, z) = 1 - \exp(-Rz) \quad . \tag{3}$$

The coefficient R is called the "sweep rate" and measures the rate at which search is carried out (see, e.g., [24]).

In order to understand (2), suppose for the moment that search allocation function f is constant over A, zero outside of A, and corresponds to a finite amount of search time T. Then for $\mathbf{x} \in A$

$$f(\mathbf{x}) = T/\text{area}\ (A) \quad ,$$

since by definition

$$T = \int_X f(\mathbf{x})\, d\mathbf{x} = f(\mathbf{x}_0) \int_A d\mathbf{x} = f(\mathbf{x}_0) \text{ area } (A)$$

for any $\mathbf{x}_0 \varepsilon A$.

The detection functional can then be written

$$D[f] = \int_X b\left(\mathbf{x},\ f(\mathbf{x})\right)\ p(\mathbf{x})\ d\mathbf{x}$$

$$= \int_A \left\{1 - \exp\left(-RT/\text{area } (A)\right)\right\}\ p(\mathbf{x})\ d\mathbf{x}$$

$$= \left\{1 - \exp\left(-RT/\text{area } (A)\right)\right\} \int_A p(\mathbf{x})\ d\mathbf{x}$$

$$= \left\{1 - \exp\left(-RT/\text{area } (A)\right)\right\}\ \Pr[\tilde{\mathbf{x}} \varepsilon A] \quad .$$

This is the so-called "random-search formula."

Sufficient Conditions for Optimal Search Plans

Using Lagrange multipliers in the fashion introduced by Everett [13], one can find sufficient conditions for optimal search plans for stationary targets that provide efficient methods for computing these plans. Define the pointwise Lagrangian ℓ as follows:

$$\ell(\mathbf{x}, z, \lambda) = p(\mathbf{x})b(\mathbf{x}, z) - \lambda c(\mathbf{x}, z) \text{ for all } \mathbf{x} \in X, \tag{4}$$

$$z \geq 0 \text{ and } \lambda \geq 0 \quad .$$

If we have an allocation f^*_λ which maximizes the pointwise Lagrangian for some value of $\lambda \geq 0$, i.e.,

$$\ell(\mathbf{x}, f^*_\lambda(\mathbf{x}), \lambda) \geq \ell(\mathbf{x}, z, \lambda) \text{ for all } \mathbf{x} \in X \text{ and } z \geq 0 \quad , \tag{5}$$

then we can show that $D[f^*_\lambda]$ is optimal for its cost $C[f^*_\lambda]$, i.e.,

$$D[f^*_\lambda] \geq D[f] \text{ for any } f \in F \text{ such that } C[f] \leq C[f^*_\lambda] \quad . \tag{6}$$

Equation (6) says that plan f^*_λ maximizes the detection probability over all plans using effort $C[f^*_\lambda]$ or less.

To show that (6) is true, we suppose $f \varepsilon F$ and $C[f] \leq C[f^*_\lambda]$. Since $f(\mathbf{x}) \geq 0$, (5) yields

$$p(\mathbf{x})b(\mathbf{x},f^*_\lambda(\mathbf{x}))-\lambda c(\mathbf{x},f^*_\lambda(\mathbf{x})) \geq p(\mathbf{x})b(\mathbf{x},f(\mathbf{x}))-\lambda c(\mathbf{x},f(\mathbf{x})) \text{ for } \mathbf{x} \varepsilon X \text{ .} \quad (7)$$

Integrating both sides of (7) over X, we obtain

$$D[f^*_\lambda] - \lambda C[f^*_g] \geq D[f] - \lambda C[f]$$

which, along with $\lambda \geq 0$ and $C[f] \leq C[f^*_\lambda]$, implies

$$D[f^*_\lambda] - D[f] \geq \lambda \, (C[f^*_\lambda] - C[f]) \geq 0 \quad .$$

This proves (6) and hence that f^*_λ is an optimal search plan for its cost $C[f^*_\lambda]$.

The way to calculate optimal search plans is to choose a $\lambda > 0$ and find f^*_λ to maximize the pointwise Lagrangian for this λ. For each $x \in X$, finding $f_\lambda(x)$ is a one-dimensional optimization problem. If the detection function is well-behaved (e.g., exponential), then one can solve for f^*_λ analytically. Since $C[f^*_\lambda]$ is usually a decreasing function of λ, one can use a computer to perform a binary search to find the value of λ which yields cost $C[f^*_\lambda]$ equal to the amount of search effort available. The resulting f^*_λ is the optimal search plan.

Example

In the case of bivariate normal target distribution (1) and exponential detection function (3), we can compute $C[f^*_\lambda]$ and the optimal plan explicitly (see [23]). The result is that for T amount of search time, the optimal allocation function f^* (dropping the subscript λ which depends on T) is given by

$$f^*(\mathbf{x}) = \begin{cases} \dfrac{1}{2\sigma^2 R} (r_0^2 - |\mathbf{x}|^2), & \text{for } |\mathbf{x}| \leq r_o \\ 0, \text{ otherwise,} & \end{cases}$$

where all search is confined to a disk of radius r_o defined by

$$r_o^{2} = 2\sigma \sqrt{RT/\pi} \quad .$$

The probability of detection corresponding to f^* is

$$D[f^*] = 1 - \left(1 + \frac{r_o^{2}}{2\sigma^2}\right) \exp\left(-r_o^{2}/ 2\sigma^2\right) \quad ,$$

and the expected time to detection $\bar{\tau}$ is

$$\bar{\tau} = 6\,\pi\,\sigma^2/K \quad .$$

Thus, for a given amount of search effort T, the optimal search plan concentrates search in the disk of radius r_o which then expands as more effort becomes available. It can be shown that the optimal search plan can be approximated by a succession of expanding and overlapping coverages. The fact that search is repeated in the high probability areas is typical of optimal search in a great many situations.

Note that probability of detection depends upon the ratio r_0/σ and increases to one as search effort increases without bound. The expected time to detection is finite and varies directly with σ^2 and inversely with the sweep rate R.

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